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The estimation of filtering rate from the clearance of suspensions

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Abstract

The indirect method of determining the filtering rate requires that a suspension feeder be allowed to feed for a given time in a volume of suspension. Six equations have been published by which the filtering rate can be calculated from the observed depletion of the suspension. By applying a standard notation throughout, it is shown that these 6 equations are identical. A graphical method is described by which the filtering rate can be obtained directly from the ratio between the initial and final concentration of suspension, obviating the need for individual calculations.

Introduction

There are 2 broad approaches to the quantitative determination of the volume of water pumped by a suspension-feeding animal. The *direct methods* rely upon the interception and measurement of the feeding current. The *indirect methods* are based upon the rate of removal of particles from a known volume of suspension. The rate so calculated is a function of the feeding current and retentivity, and is conveniently termed the 'filtering rate' to distinguish it from the 'pumping rate' measured by the direct methods (Coughlan and Ansell, 1964).

Dodgson (1928) appears to have been the first to attempt to calculate the filtering rate from the rate of removal of particles from suspension. His values were minimal, since he was unable to compensate mathematically for the progressive dilution of his suspensions. Subsequently, at least 6 equations have been published which enable the filtering rate to be estimated reliably from the rate of change of the concentration of particles in suspension. Four assumptions are fundamental:

a) The reduction in the concentration of particles is due to filtration by the animal, and to gravitational settling.

b) The animal's pumping rate is constant.

c) Particle retention is 100% efficient; alternatively a known, constant percentage is retained.

d) The test suspension is at all times homogeneous.

Given these conditions, the filtering rate calculated would be the volume of water actually pumped by the animal in a given period of time.

In a clearance experiment the animal is continuously withdrawing particles and diluting the suspension with filtrate. If the filtering rate remains constant, then the rate at which particles are removed will progressively decline, i.e. as the concentration decreases, so do the decrements; successive decrements bearing a fixed ratio to the existing concentration. Decrements are removed at indefinitely small intervals so that concentration is a continuous function, as represented by the curve e^{-x}.

The general equation may be written

$$\frac{dC}{dt} = -C\left(\frac{mn}{M} + a\right) \tag{1}$$

where $\frac{dC}{dt}$ is the rate of decrease of concentration 'C' at time 't'; 'M' is the volume of suspension; 'n' is the number of animals; 'm' is the filtering rate of a single animal; and 'a' is the rate at which particles settle out of suspension.

The general solution of (1) is

$$C_t = C_0 \cdot \exp^{-\left(\frac{mn}{M} + a\right) \cdot t}$$

which in logarithm form becomes

$$\log_e C_0 - \log_e C_t = \left(\frac{mn}{M} + a\right) \cdot t$$

or

$$m = \frac{M}{n}\left[\frac{(\log_e C_0 - \log_e C_t) - a}{t}\right] \tag{2}$$

'a' can be determined from a control experiment with no animals present

$$C_t' = C_o' \cdot \exp^{-a \cdot t}$$

or

$$a = \frac{\log_e C_o' - \log_e C_t'}{t} \tag{3}$$

where C_0' is the initial concentration and C_t' the concentration after time 't'.

The 6 published equations can be shown to be identical to equation (2) above. Any apparent dissimilarity is due to differing notation, to the omission of terms following standardisation of an author's method, and to a general preference for common, rather than natural, logarithms.

Equations

One of the first to be published was that of Fox et al. (1937), of the form:

$$m = M\,\frac{b - a}{n \cdot t} \tag{4}$$

where 'm' is the filtering rate; 'M' is the volume of suspension; 'a' is the logarithmic decrement in the control (i.e. $\log_e \mathrm{conc}_o' - \log_e \mathrm{conc}_t'$; conc_o' and conc_t' being the concentration of the suspension initially and after time 't'); 'b' is the logarithmic decrement in the test suspension ($\log_e \mathrm{conc}_o - \log_e \mathrm{conc}_t$); '$n$' is the number of animals per test vessel; and 't' is the duration of the experiment (supplied by the present author since Fox et al. used a 1 h period throughout). This is identical to equation (2).

FULLER (1937) published an equation for the clearance of a suspension by copepods:

$$W_X = V \cdot \log_e \frac{\text{conc}_0}{\text{conc}_t} \tag{5}$$

where 'W_X' is the volume swept clear by each copepod in 'x' hours; 'V' is the volume available to each copepod; other symbols as before. Reducing to unit time:

$$W = \frac{V}{X} \log_e \frac{\text{conc}_0}{\text{conc}_t} \tag{6}$$

where 'W' is the volume swept clear per hour. Applying our previous notation and introducing 'n'

$$m = \frac{M}{n \cdot t} (\log_e \text{conc}_0 - \log_e \text{conc}_t) \tag{7}$$

which lacks 'a' but is otherwise identical to (2).

Similarly with JÖRGENSEN's formula (1943), which is usually written

$$m = M \cdot \frac{(\log_{10} \text{conc}_o - \log_{10} \text{conc}_t)}{\log_{10} e \cdot t} \tag{8}$$

the logarithm base can be changed for any value:

$$m = M \cdot \frac{(\log_e \text{conc}_0 - \log_e \text{conc}_t)}{\log_e e \cdot t} \tag{9}$$

however '$\log_e e$' is unity. If 'n' is introduced to cover the use of more than 1 animal, this equation is identical to equation (7).

WILLEMSEN (1952) derived an equation from first principles, after the method of FOX et al.

$$x = m \frac{(\log_e \text{conc}_0 - \log_e \text{conc}_t - a)}{t} \tag{10}$$

where 'x' is the filtering rate; 'm' is the volume of suspension; other symbols as before. With the inclusion of 'n', and changed to standard notation, this equation becomes identical with (2). QUAYLE (1948) considered the process as negative compound interest, and produced an equation:

$$m = \frac{M}{n \cdot t} \left[\left(\log_e \frac{\text{conc}_0}{\text{conc}_t} \right) - \left(\log_e \frac{\text{conc}_{0'}}{\text{conc}_{t'}} \right) \right] \tag{11}$$

where 'conc_o', 'conc_t' are the concentrations initially and after time 't' in the test suspension; '$\text{conc}_{o'}$', '$\text{conc}_{t'}$' those in the control suspension. Other symbols as before. It is evident that the ratios can be obtained by subtracting their respective logarithms, thus:

$$m = \frac{M}{n \cdot t} [(\log_e \text{conc}_0 - \log_e \text{conc}_t) - (\log_e \text{conc}_0{}' - \log_e \text{conc}_t{}')] \tag{12}$$

which is identical to (2).

Finally, THEEDE (1963) gives

$$f = 1{,}2303 \frac{(\log_{10} \text{conc}_0 - \log_{10} \text{conc}_t)}{(t - t_o) \cdot n} \tag{13}$$

where $(t - t_o)$ is the time lapse; 'f' is the pumping rate, ('t' and 'm' respectively in our notation). 1,2303 can be recognized as the numerical value of $\log_e 10$, which is the reciprocal of $\log_{10} e$. THEEDE habitually used 1,0 l of suspension, so it is necessary to introduce 'M'. By these substitutions the expression becomes

$$m = M \cdot \frac{(\log_{10} \text{conc}_0 - \log_{10} \text{conc}_t)}{\log_{10} e \cdot t \cdot n} \tag{14}$$

and is identical to equation (8).

By employing a standard notation and introducing additional terms where necessary, the 6 published equations are shown to be identical, thus justifying direct comparison of the results obtained by their use.

In practice, QUAYLE's equation (11) (1948) was found simplest to apply. Where settling is negligible, the final term can be omitted

$$m = \frac{M}{n \cdot t} \cdot \log_e \frac{\text{conc}_0}{\text{conc}_t} \tag{15}$$

Graphical method

In a series of determinations of filtering rate it is often found that a standard volume of suspension, and standard intervals of time, are being used habitually. It is then possible to simplify the mathematical

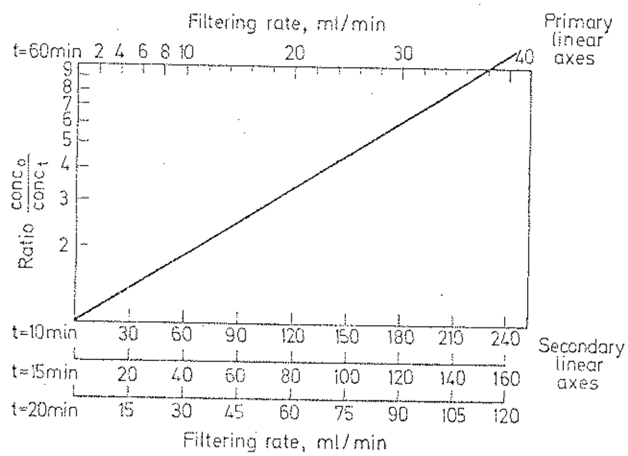

Fig. 1. *Construction:* Calculate the filtering rates corresponding to 3 arbitrary values of the ratio $\text{conc}_0/\text{conc}_t$; using $m = \frac{M}{n \cdot t} \log_e \frac{\text{conc}_0}{\text{conc}_t}$ with $M = 100$ ml, $t = 60$ min, $n = 1$. The secondary axes are set out by proportion.

Use: Calculate the ratio $\text{conc}_0/\text{conc}_t$; which will be at least unity. Find this position on the line; read off filtering rate from whichever "t" scale is applicable

procedure by first calculating the filtering rates obtained when 3 arbitrary values of the ratio $\text{conc}_0/\text{conc}_t$ are substituted in equation (15). 'M' is the standard volume, and 't' (for convenience) 60 min. These 3 calculated rates are plotted against the corresponding ratios on log/linear paper (Fig. 1). Additional scales corresponding to any required time interval are set out along the linear axis by simple proportion. Filtering rates can then be obtained directly from the ratio by reading off the appropriate scale.

When gravitational settling cannot be ignored, the settling rate is obtained from the same graph using the ratio $\text{conc}_0'/\text{conc}_t'$ (control suspension). This rate is then deducted from the *apparent* filtering rate obtained from the ratio $\text{conc}_0/\text{conc}_t$ (test suspension) to give the corrected rate.

Summary

1. Six equations, published over a period of 25 years, with the common purpose of enabling filtering rate to be estimated from the clearance of suspensions, are shown to be identical.

2. A graphical method for the rapid estimation of filtering rate is described. This is of particular benefit in a series of determinations where volumes and time-intervals are standardised.

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